

Summary

In part 1 we looked at the four basic operators:

- addition
- subtraction
- multiplication
- division

In part 2 we learnt about more tricky combinations of the four operators and how the natural order of precedence can be used to find the correct answer.

In part 3 we will introduce the idea of brackets and learn how they can be used to change the natural order of precedence.

Brackets

Brackets are used to force the terms inside the brackets to be calculated first.

Example 1

This is based on the example we used in part 2:

We asked: What is the answer to $8 - 3 + 6$?

The correct answer is 11. We then showed the wrong way to calculate the answer: the result of that calculation was -1 .

Now is the time to show why it was wrong.

We can use brackets in two ways:

$$(8 - 3) + 6 \quad \text{or} \quad 8 - (3 + 6)?$$

Method 1	Method 2
Step (a): subtract the "3" from the "8": $8 - 3 = 5 \dots\dots\dots (a)$	Step (a): add the "3" to the "6": $3 + 6 = 9 \dots\dots\dots (a)$
Step (b): add "6" to the result: $5 + 6 = 11 \dots\dots\dots (b) \checkmark$	Step (b): subtract the result from "8": $8 - 9 = -1 \dots\dots\dots (b) \checkmark$

The answer to method 1 is correct because we followed the natural order of precedence. The brackets in method 1 do not change the natural order of precedence.

The answer to method 2 is also correct – but only because the brackets change the natural order of precedence.

Example 2

In part 2 we asked: What is the answer to $3 + 5 \times 7 + 8$?

When we add brackets we change the question completely:

What is the answer to $(3 + 5) \times (7 + 8)$?

There is only one correct way to work this out:

Step (a): first add the "3" and the "5" in the first pair of brackets:

$$3 + 5 = 8 \dots\dots\dots (a)$$

Step (b): next add the "7" and the "8" in the second pair of brackets:

$$7 + 8 = 15 \dots\dots\dots (b)$$

Step (c): multiply the two intermediate results from (a) and (b):

$$8 \times 15 = 120 \dots\dots\dots (c) \checkmark$$

Example 3

What is the answer to $8 - (3 + 6)^2$?

This time the brackets demand we find the sum of $3 + 6$ before we find the square:

Step (a): first add the "3" and the "6" inside the brackets:

$$3 + 6 = 9 \dots\dots\dots (a)$$

Step (b): next calculate the square of "9":

$$9^2 = 81 \dots\dots\dots (b)$$

Step (c): subtract the intermediate result (b) from "8":

$$8 - 81 = -73 \dots\dots\dots (c) \checkmark$$

Example 4

In part 2 we asked: What is the answer to $12 \div 3 \times 6$?

Once again, when we add brackets we change the question completely:

What is the answer to $12 \div (3 \times 6)$?

Step (a): first multiply the "3" by the "6":

$$3 \times 6 = 18 \dots\dots\dots (a)$$

Step (b): then divide the "12" by the result:

$$12 \div 18 = \frac{2}{3} = 0.667 \dots\dots\dots (b) \checkmark$$

Example 5

In part 2 we asked: What is the answer to $63 + 20 \div 4 - 12 \times 2$?

The answer is 44.

But what happens if we change the question by adding brackets?

What is the answer to $(63 + 20) \div (4 - 12) \times 2$?

The brackets demand that we deal with the items they contain first.

Step (a): first add the “63” to the “20”:

$$63 + 20 = 83 \dots\dots\dots (a)$$

Step (b): subtract the “12” from the “4”:

$$4 - 12 = -8 \dots\dots\dots (b)$$

Rewrite the original calculation using our intermediate results (a) and (b):

$$\begin{aligned} & (63 + 20) \div (4 - 12) \times 2 \\ & = 83 \div (-8) \times 2 \\ & = -83 \div 8 \times 2 \\ & = -83 \div 16 \\ & = -5.1875 \end{aligned}$$

Example 6

What is the answer to the following?

$$3 + \sqrt[2]{4 \times (7 + 9)}$$

Once again, brackets force the order of precedence so the answer is:

$$\begin{aligned} & = 3 + \sqrt[2]{4 \times 16} \\ & = 3 + \sqrt[2]{64} \\ & = 3 + 8 = 11 \end{aligned}$$

Notice how we have started each stage of the calculation with an “=” sign and that all the “=” signs line up vertically under each other. Setting out your work neatly like this is all part of your “presentation” skills.

Example 7

Another more complex example:

$$\frac{7-3}{2} + 12 \times 4 - 2$$

There is more than one way to simplify this calculation. The equation has implied brackets as follows:

$$\frac{(7-3)}{2} + 12 \times 4 - 2$$

So far you have learnt to deal with the terms inside the brackets first. This isn't always necessary – but you do have to keep those terms together until you do deal with them. Two different ways have been shown side-by-side for you to compare first dealing with the brackets (method 1) and second, leaving the brackets till later (method 2).

Method 1	Method 2
<p>Here we deal with the “7-3” in the numerator to simplify the calculation. We make the subtraction $7 - 3 = 4$:</p> $= \frac{4}{2} + 12 \times 4 - 2$ <p>Make the division: $4 \div 2 = 2$:</p> $= 2 + 12 \times 4 - 2$ <p>Now we can deal with the multiplication:</p> $= 2 + (12 \times 4) - 2$ $= 2 + 48 - 2$ $= 50 - 2 = 48 \checkmark$	<p>Here we deal first with the multiplication of $12 \times 4 = 48$:</p> $= \frac{7-3}{2} + (12 \times 4) - 2$ $= \frac{7-3}{2} + 48 - 2$ <p>Make the subtraction $48 - 2 = 46$:</p> $= \frac{7-3}{2} + 46$ <p>Finally we deal with the fractional part by first subtracting $7 - 3 = 4$:</p> $= \frac{4}{2} + 46$ <p>Make the division $4 \div 2 = 2$ and add to the “46”:</p> $= 2 + 46 = 48 \checkmark$

This is the last example in the series on operators. All you need to do now is go and practise them!